Culbertson (H.)

REFRACTION OF THE EYE,

AS DISTINGUISHED FROM ACCOMMODATION AND ESTIMATED AS AN EQUIVALENT, FROM THE INDEX OF REFRACTION.

—BY—

H. CULBERTSON, M.D., Asst. Surgeon, U.S.A., Retired.

The distinction between the *refraction* of the eye in a state of activity from the force of the ciliary muscle, and *that* observed while this organ is at rest, has been drawn by Donders and other authorities. The object of this paper is to estimate this difference, as an equivalent, by the index of refraction.

All know that one of the essentials to perfect vision is the presence of refraction, both in active and passive sight; but how much refraction is due to the passive (from simple *capacity*) condition, and what proportion to the active state of the ciliary muscle, is a subject as important as it is interesting. The purpose is to treat this *capacity* and this *force* as distinct factors in vision.

For refraction in a state of *rest* the symbol Re will be employed, in order to distinguish it from small "r", or "punctum remotum", or infinity, or ∞.

For that refraction which is the result of the action of the ciliary muscle applied upon the crystalline lens, the symbol C will be used, meaning Len-cil-tasis; from Lentis, lens; cilia, hairs, (the ciliary muscle being named from its ciliated feature), and tasis, tension: By which is understood the force of the ciliary muscle applied upon the lens in vision. The symbols denoting "*accommodation*" "A", and "*astigmatism*" "A", are thus avoided, as well as the confusion from their use.

The hypothesis is that visual power—V P, or V = Re + C; and hence Re = V P — C; or C = V P — Re.

The influence of Re is due to refraction of the humors, the lens and cornea when the eye is at rest. Through Re parallel rays of light, or those from infinity, and incident upon the cornea are brought to a focus upon the layer of rods and cones of the retina. Practically, five metres, or 500 centimetres (cm.), or 5000 millimetres, (mm.), or 200″ inches (″) will be the equivalent for infinity, (∞). The metrical system and the English inch will be employed in the calculations.

By the metrical system is understood a *unit* of linear measure of one (1) metre, which is equal to 100 centimetres (cm.) or 1000 millimetres (mm.) or 40″ english inches, (″). This unit is divided into 20 parts each of which is a "dioptry" or "dioptric"; symbol D. Thus:

D 1.0,	= 1	metre, or	100	cm. or	40″	inches;
D 2.0,	= $\frac{1}{2}$	" "	50	" "	20″	"
D 3.0,	= $\frac{1}{3}$	" "	$33\frac{1}{3}$	" "	$13\frac{1}{3}$″	"
D 4.0,	= $\frac{1}{4}$	" "	25	" "	10″	"
D 5.0,	= $\frac{1}{5}$	" "	20	" "	8″	"
D 6.0,	= $\frac{1}{6}$	" "	$16\frac{2}{3}$	" "	$6\frac{2}{3}$″	"
D 7.0,	= $\frac{1}{7}$	" "	$14\frac{2}{7}$	" "	$5\frac{5}{7}$″	"
D 8.0,	= $\frac{1}{8}$	" "	$12\frac{1}{2}$	" "	5″	"
D 9.0,	= $\frac{1}{9}$	" "	$11\frac{1}{9}$	" "	$4\frac{4}{9}$″	"
D 10.0,	= $\frac{1}{10}$	metre, or	10	cm. or	4″	inches.
D 11.0,	= $\frac{1}{11}$	" "	$9\frac{1}{11}$	" "	$3\frac{7}{11}$″	"
D 12.0,	= $\frac{1}{12}$	" "	$8\frac{1}{3}$	" "	$3\frac{1}{3}$″	"
D 13.0,	= $\frac{1}{13}$	" "	$7\frac{9}{13}$	" "	$3\frac{1}{13}$″	"
D 14.0,	= $\frac{1}{14}$	" "	$7\frac{1}{7}$	" "	$2\frac{6}{7}$″	"
D 15.0,	= $\frac{1}{15}$	" "	$6\frac{2}{3}$	" "	$2\frac{2}{3}$″	"
D 16.0,	= $\frac{1}{16}$	" "	$6\frac{1}{4}$	" "	$2\frac{1}{2}$″	"
D 18.0,	= $\frac{1}{18}$	" "	$5\frac{5}{9}$	" "	$2\frac{2}{9}$″	"
D 20.0,	= $\frac{1}{20}$	" "	5	" "	2″	"
D 0.75,	= $1\frac{1}{3}$	" "	133	" "	$53\frac{1}{3}$″	"
D 0.5,	= 2	" "	200	" "	80″	"
D 0.25,	= 4	" "	400	" "	160″	"
D 0.00,	= 5	" "	500	" "	200″	"

Dioptrics are converted into English inches, by dividing one (1) metre or 40″ inches, by the number of the dioptry. Thus $D = 2.0 = \frac{40''}{D\,2} = 20''$ inches.

With a metrical measure the ametropia is readily estimated. Thus if the focal distance of the glass required is 50 cm., then as $2\frac{1}{2}$ cm. is equal to one English inch $\frac{50 \text{ cm.}}{2.5''} = 20''$ inches.

The focal distance can also be determined by dividing 100 cm. or 1 metre, by the number of cm. found denoting the focal distance of the glass required, which will convert the cm. into dioptrics, and if 40″ or one (1) metre be divided by the number of dioptrics found, the quotient will be the focal distance of the glass required in English inches. This system has its advantages and is adopted by many ophthalmologists.

Forty inches is however a fraction more than one (1) metre, the latter being 39.371+ English inches. Practically the former quantity is regarded as accurate.

These explanations seem necessary for the general medical reader, as are the following illustrations important in relation to this subject.

Fig. 1, represents Helmholtz diagrammatic eye; Fig. 2, the posterior focus for rays of light parallel in the air, and Fig. 3, the anterior focus for rays parallel in the vitreous humor.

FIG. 1.

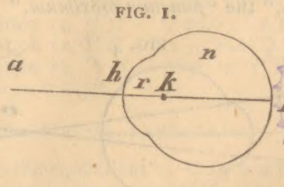


FIG. 2.

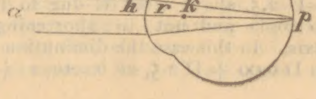
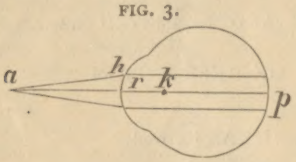


FIG. 3.



Let "a", Fig. 1, be the *anterior* focal point; "p", the *posterior* focal point; "h", the *principal* point; "a h" = F′, the anterior *focal distance* of the eye = 15.4983 mm.; " h p" = F″, the posterior focal distance = 20.7135 mm.; "n" the index of refraction of the eye = 1.3365 +; "k" the nodal point; "h k" = "r" the radius of curvature of the cornea = 5.2152 mm., of the diagrammatic eye.

These are the measurements of Heyl, who is supported in his estimates by Stammehaus and Reich (see p. 91, vol. viii, Arch. Ophth).

It is evident that rays of light emanating from an infinite distance, or from 5000 mm. from "k" Fig. 1, will be parallel to the axis "a p", and be brought to a focus at "p"; and as the index of refraction for the eye is 1.3365, such a degree of refraction is sufficient to induce the focus at "p", of rays from 5000 mm. distance; it follows that a co-efficient of refraction of 1.3365 is equivalent to 5000 mm. As this is the result altogether of refraction, or Re, it ensues that 5000 mm. is equal to Re.

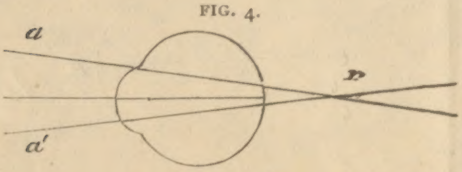
In vision of objects 5000 mm. from "k," the ciliary muscle is at rest, and hence Re unaided is sufficient to focus parallel rays at "k." But within this distance C begins to act. The 5000 mm. is equal to dioptrics 0.00. Hence C at 5000 mm. is = D 0.00, or at that distance there is no C exercised in the emmetropic eye.

The following results:

Proposition 1st, Em Re = D. 0.00 = 1.3365.
Proposition 2d, Em C = D. 0.00 = 0.000.

Re remains D 0.00 up to fifty years, when it begins to decline, until at eighty years, it is reduced = D 2.5 (negative). That is the far point is no longer at D 0.00 (200″) in front of "k," but lies theoretically—D 2.5, or 16″ posterior to that point (k). The posterior focal distance for rays *convergent* in front of the cornea is located at the point where such rays decussate posteriorly, and is the "*punctum proximum*" in the case. This is illustrated by Fig. 4, in which a a′ represent converging rays of light, "a" prolonged in "r," and a′ in "r," at which point "r" is "p," the "*punctum proximum.*"

FIG. 4.



Such an eye cannot command parallel, much less divergent rays of light, when at rest.

The—D 2.5, above cited is due to refractive hypermetropia and not to shortening of the visual axis. In this case the diminution of Re is equal to D 0.00 + D 2.5, or is=to a + lens of 16″ focal distance (D 2.5 = 16″). What is Re = —D 2.5 in terms of refraction? This may be estimated, as for axial hypermetropia and in the same manner as if the visual axis were shortened.

The formula deduced from Schwigger (Handbook of Ophthalmology, p. 103) will be employed: viz.: $\frac{1}{F} = \frac{1}{F_{''}} + \frac{1}{no}$; in which $\frac{1}{F}$ = the conjugate focal distance sought; $\frac{1}{F_{''}}$ = the posterior focal distance of the diagrammatic eye = 20.7135 mm.; n = the index of refraction of the emmetropic eye = 1.3365; and o = the distance of the object from the nodal point, k, or, in this case, 400—5.2152 = 394,7848 mm. from p. (Fig. 1). Substituting $\frac{1}{F} = \frac{1}{20.7135} + \frac{1}{394.7848 \times 1.3365}$ = 19.9310 mm., which is the posterior focal distance of such an eye. The index of refraction in the case will be $\frac{19.9310}{15.4983}$ = 1.2860. Then Re = — D 2.5 is equal to 1.2860. At fifty years of age the index of refraction, the Re, of the eye is 1.3365, while at eighty years, it has diminished to 1.2860, a reduction of Re = 1.3365 — 1.2860 = 0.0505, hence the following:

Proposition 3, H. R. = — D 2.5 = 1.2860.
Proposition 4, In H. = — D 2.5, Re is defective 0.0505.

Axial Hypermetropia.

In this form of H, the ametropia is due to shortening of the visual axis of the eye. The relation between the axis and Re is destroyed, and both are diminished. Hence the degree of H may be estimated from the shortening of this axis, and the curtailed visual axis will be the posterior focal distance.

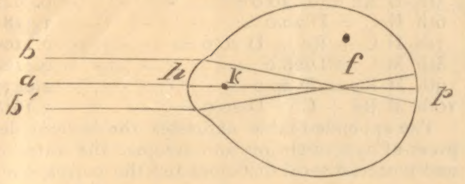
The formula employed in the calculation, will be that above given, viz: $\frac{1}{F} = \frac{1}{F_{''}} + \frac{1}{no}$, and the problem is, what is the Re in a H = D 20.0, or in a H of $\frac{1}{2}$? Here let $\frac{1}{F}$ = the conjugate focus sought; $\frac{1}{F_{''}}$, = the posterior focal distance of the emmetropic eye = 20.7135 mm.; n. = the index of refraction = 1.3365, o, = the distance of the object from "k," (Fig. 1) or from "h," = 50 — 5,2152=44.7848 mm. substituting, $\frac{1}{F} = \frac{1}{20.7135} + \frac{1}{44.7848 \times 1.3365}$ = 15.3882 mm., the posterior focal distance in the case. The index of refraction = $\frac{15.3882}{15.4983}$ = 0.9928; which is a reduction of Re = 1.3365 — 0.9928 = 0.3437; Hence,

Proposition 5, H. Re = D 20.0 = 0.9928.
Proposition 6, In H. = D 20.0, Re is defective, 0.3437.

Axial Myopia.

In this form of ametropia the visual axis is increased and the retina, now receded from the emmetropic position, 20.7135 mm. from "h," (Fig. 5), the rays of light decussate at f (Fig. 5), and are spread upon the retina, at "p" in circles of diffusion. This may be better understood by reference to Fig. 5, in which b b′, represent parallel rays of light incident upon the cornea. From the RELATIVE *excess* of the refraction of such an eye, the rays b b′ are brought to a focus at "f" and pass posteriorly to the retina at "p",

FIG. 5.



The ametropia is here determined by estimating the posterior focal distance or the length of the visual axis. The formula employed will be that given by Schwigger (Handbook Ophth., p. 103), viz: $\frac{1}{F} = \frac{1}{F_{\prime\prime}} - \frac{1}{no}$, in which $\frac{1}{F}$ = the length of the visual axis desired; $\frac{1}{F_{\prime\prime}}$, = the posterior focal distance of the diagrammatic eye = 20.7135 mm.; "n", the index of refraction = 1.3365; and "o" the distance of the object from k — (h. k.) = 50 — 5.2152 = 44.7848 mm. Substituting, $\frac{1}{F} = \frac{1}{20.7135} - \frac{1}{44.7848 \text{x} 1.3365} = 31.6750$ mm. which is the posterior focal distance for 20 D. The index of Re will be found by dividing the posterior focal distance found, by the normal anterior focal distance, or $\frac{31.6750}{15.4983} = 2.0437$.

The excess of Re in this case is = 2.0437 — 1.3365 = 0.7072: hence

Proposition 7, M. Re = D 20.0 = 2.0437.

Proposition 8, In M. = D 20.0, Re is in excess, 0,7072.

Thus far the eye has been considered in a state of rest. It remains to estimate what is the equivalent in refraction, denoting the force of the ciliary muscle in the range of vision, or what is the Lenciltasis or C.

Since it was found that C begins within 5000 mm. from "k" (Fig. 1) the range of Lenciltasis extends from that point (5000 mm.) towards the eye, and experiment and observation have shown that the average near point is about 70 mm. from "h", (Fig. 1) or 7 cm. or 2.8″. The range of Lenciltasis (C) is equal to D 14.0 — D 0.0 = D 14.0. What is the index of refraction equivalent to D 14.0? One of the steps necessary in determining this is to estimate the range of Lenciltasis C, by the well-known formula of Donders, viz: $\frac{1}{C} = \frac{1}{p} - \frac{1}{r}$, "p" and "r" being calculated from the nodal point, "k", 7 mm. behind "h", as determined by that authority, (see p. 79, Anom., Acc. and Refrac.) for the *reduced* diagrammatic eye. Then "p" = D 14.0 or 70 + 7 = 77 mm., and "r" = 5000 + 7 = 50007 mm. Substituting $\frac{1}{C} = \frac{1}{77}, - \frac{1}{5007} = \frac{4930}{385489}$ = the range of C. As Donders estimates the refraction induced by the increased curvature of the crystalline lens, or C, at ninth-tenths ($\frac{9}{10}$), the last quantity found should be $\times$ 0,9, which will give the focal distance of the auxiliary lens required in the case. Then $\frac{4930}{385489} \times \frac{9}{10} = \frac{1}{84.6}$, or a + lens of 84.6 mm. about 3.3″ inches principal focal distance. What is the equivalent of this lens in refraction, or in C? In estimating this it must be remembered that an object situated at 84.6 mm. from the cornea would subtend divergent rays of light, which would focus behind the retina did not the lens become more convex through C and cast the focus upon the retina in its normal position. It is evident that this theoretical negative position of the focal point behind the retina (which would be the posterior focal point were Re alone exercised and C not employed) bears the same relation to the position of the normal retina, as does the position of the normal retina to the shortened visual axis, or the advancement of the posterior focal point, in axial hypermetropia. Hence the formula for determining the shortening of the visual axis in H, will give the solution in this case, viz: $\frac{1}{F} = \frac{1}{F_{\prime\prime}} + \frac{1}{no}$, in which $\frac{1}{F}$ = the posterior focal distance sought; $\frac{1}{F_{\prime\prime}}$ = posterior focal distance of the diagrammatic eye = 20.7135 mm.; "n", = the index of refraction of the normal eye = 1.3365 and and "o", the distance of the object k — (h k) (Fig. 1) = 84.6 — 5.2152 = 79.3848 mm. Substituting, $\frac{1}{F} = \frac{1}{20.7135} + \frac{1}{79.3848 \text{x} 1.3365} = 17.3301$ mm., which would be the posterior focal distance for parallel rays for the hypermetropic eye. This quantity divided by the anterior focal distance, will give the index of refraction equivalent to the auxiliary + lens of 84.6 mm. focal distance, viz: $\frac{17.3301}{15.4983} = 1.1181$, which is equivalent, as an index, to the *ciliary-muscle-force*, the C, exercised when the object is situated at 84.6 mm. from k (Fig. 1).

Visual power will equal the Em Re + this quantity, or 1.3365 + 1.1181 = 2.4546, for Re + C = D 14.0; hence

Proposition 9, Em C. = D 14.0 = 1.1181.

Proposition 10, Em C. + Re = 14.0 = 2.4546.

Hypermetropic Lenciltasis.

What is the C, in a hypermetropia = D 20.0?

In this case the near point would be, were it possible, 50. mm., behind the nodal point k. The punctum remotum .r, lies still further back of the eye. It has been seen that the index of refraction for a hypermetropia = 20.0 D, is 0.9928, and that in this degree of H, there is a defect of Re = 0.3437. The total Em C = 1.1181. Hence in a H = D 20.0, the C + Re, or 1.1181 + 0.9928 = 2.1109. This defect, 0.3437, is, 0.3437 more C than the ciliary muscle can exexercise. But to compensate for this defect the ciliary muscle must exercise a force = 1.1181 + .3437, = 1.4618, which is 0.3437 more C than it is capable of exercising. The V P is defective because Re is reduced to 0.9928 index, hence:

Proposition 11, H C = D 20.0 = 1.1181.

Proposition 12, H C + Re = 20.0 D. = 2.1109.

Proposition 13, In H = D 20 0, V P is defective in Re = 0.3437.

Myopic Lenciltasis.

What is the equivalent of C in a myopia = D 20.0, the near point being 50 mm. from the nodal point k? It was found that in a myopia of this degree that Re was equivalent to 2.0437, and Em C, to 1.1181. Re is in excess in the case = 2.0437 —1.1181 = 0.9256. As the 2.0437, includes the 1.3365 of emmetropic Re when the eye is at rest, then if the near point be at 50 mm, from k, the sum of Re + C will equal 2.0437 +

1.1181 = 3.1618, if C be all exercised in a M = D 20.0. Em Re + C being equal to 2.4546, it results in a myopia of this degree, that there is an excess of Re + C = 3.1618 — 2.4546 = 0.7072. As this quantity cannot be taken from the Re, it must be deducted from the C in the case, or 1.1181 — 0.7072 = 0.4109 C employed in the case, Hence:

Proposition 14, M C = D 20.0 = 0.4109.

Proposition 15, M C + Re = D 20.0 = 3.1618.

Proposition 16, In M = D 20.0, C is in excess, 0.7072.

Proposition 17, M Re = D 20.0 = 2.0437.

CONCLUSIONS.

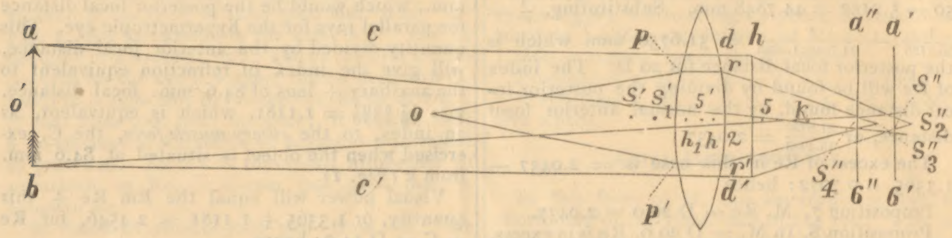
1st	Em Re = D 20.0 =	1.3365
2d	Em C = D 20.0 =	1.1181
3d	Em Re + C = D 20.0	2.4546
4th	H Re = D 2.5 =	1.2860
5th	H Re = D 20 0 =	0.9928
6th	H C = D 20.0 =	1.1181
7th	H C + Re = D 20.0 =	2.1109
8th	M C = D 20.0 =	0.4109
9th	M Re = D 20.0 =	2.0437
10th	M Re + C = D 20.0 =	3.1618

The appended table expresses the several degrees of hypermetropia and myopia, the anterior and posterior focal distances and the corresponding indices of refraction for the several grades of ametropia, as calculated from the above formula.

An example illustrating the practical application of this method will be given, viz: the determination of the proper test types, estimated by Stammehaus and Heyl constants, as well as to decide the position and influence of the correcting lens in a case of hypermetropia = 20 dioptrics.

Fig. 6 is necessary.

FIG. 6—LENS.



The *test types* of Snellen, estimated by the measurements given by Donders for the diagrammatic eye, for 5000 mm. from k are, 7,3359 mm. in height. The height of the image at S″ is .0218 mm. in this calculation.

With the measurements as given by Heyl the types for the same distance are smaller, being 7.0330 mm. or 7.3359 — 7.0330 = .3029 mm. less, which is a difference too great to be ignored.

These results are produced from the formula, viz: k S″ : S″ : : k 6 : a b, (see fig. 6) substituting Heyl's constants, 15.4983 : .0218 : : 5000 : 7.0330 : or with Donders' constants 14.8583 : .0218 : : 5000 : 7.3359 mm.

In considering the case of hypermetropia of 20 dioptrics, the following is assumed:

See fig. 6, h k, from cornea to nodal point = 5.2152 mm;

h S″ cornea to retina = 20.7135 mm.

h S′ cornea to anterior focal point = 15.4983 mm.

h S'_1 cornea to anterior locus = 13.7451 mm.

h S''_2 cornea to posterior locus = 22.8236 mm.

h S''_4 posterior axis for an eye with 20 dioptrics of hypermetropia = 15.3882 mm.

The + correcting lens of 50 mm. focal distance from "5′". Its thickness 7.5 mm., its aperture (diameter) 36.5 mm., its principal points h_1 h_2, at 1 mm. from the anterior and posterior surface, and its position 5′, at 13.7451 from k; what then will be the posterior locus, or the distance S''_3 S''_4? To determine this we deduct the place of the lens, from 50 mm. the power (focus) of the + correcting lens, and adopt the following formula:

50—13.7451 = 36.2549 mm. Hence:

$\frac{36.2549 \times 15.3882}{36.2549 + 15.3882}$ = 10.8029 mm. from k, or 10.8029 + 13.7451 = 24.5480 + 2.1101 (the emmetropic locus of the normal eye), = 26.6581 mm. from 5′.

Now 13.7451 — 5.2152 (distance from h to k) = 8.5299 mm. + 15.3882 mm. (distance from h to S''_4 = 23.9181 mm., and 26.6581 (5′ to S''_3 — 23.9181 (5′ to S''_4) = 2.7400 mm = the posterior locus S''_3 to S''_4, in the case = 5′ to h_2 of the lens.

The lens, to correct the ametropia being applied, as above, where will the optical centre or 5 be found within the eye. The distance between, h k, must be divided into two parts proportional to the anterior and posterior focal distances, or S′ 15,4983 mm., and S''_4 15.3882 mm., and the effect of the correcting lens must cause k to be brought to 5. Hence 15.4983 — 15.3882 = .1101 mm., and 15.3882 × .1101 = 1.69424082 ÷ (15.4983 + 15.3882 =) 30.8865 = .0548 mm. behind h; or 15.3882 — .0548 = 15.3334 mm in front of S''_4.

What test types will be required when the image at S''_4 on the line a″ 6″ will = .0218 mm. in height. Formula, 5 S''_4 (15.3334) mm. : .0218 : : 5 6, or (5.2151 — .0548 = 5.1604 from 5000) = 4994.8396 mm. : 7.1013 mm., = a b at 5000 mm. from k, when the correcting lens is applied.

Thus to produce .0218 mm. at S''_4 the types must be, at 5000 mm. from k, 7.1013 mm. in height.

The types for this distance are smaller for the emmetropic eye, estimated by the constants of Heyl, = 7.1013 — 7.0330 = .0683 mm. less.

This difference may be disregarded considering the distance of the types from k. If however the types of Snellen be compared with the above,

there is a marked difference, = 7.3359 (Snellen) — 7.0330 = .3029 mm. a quantity too great to be ignored.

With the correcting lens applied to the eye as above, the angle under which S''_4 is seen is 4′ 51″ and the angle under which S″ is viewed is 4′ 48″, that is when the value at a″ 6″ and a′ 6′ is = .0218 mm., which latter quantity produced or would be seen under, with Donders constants, an angle of 5′.

What will be the value of S''_4 when a b test type = 7.0330, the correcting glass applied, and Heyl's constants employed? Formula: 4994.8396 : 7.0330 : : 15.3334 : .0215 mm., which latter is a smaller image than is necessary to be able to see its several parts under an angle of 60″. To remedy this the types must be enlarged. Under what angle will such an image be seen. Formula $\frac{.0215 \times 180^\circ}{15.3334 \times 3.1416} = \frac{3.8700}{48.17140944} = .080^\circ = 4' 48''$.

The types being increased to 7.1013, at 5000 mm. from k, the image will become .0218 mm. at S''_4, and the angle under which the image is seen 4′ 51″, which is equivalent to an angle of 5′, under the constants of Donders.

It may now be inquired if the test types can be estimated from the refraction of the eye considered as an equivalent. In this estimate the following course is adopted. The normal refraction 1.3365 is the basis of calculation. This quantity, as is well-known, is derived from the relation between the anterior and posterior, principal focal distances of the emmetropic eye. It results that, if we could construct an eye with a posterior focus of 1, the anterior focus would be .3365 under the proposition. Hence (see Fig. 6) as h″ S″ = (20.7135 mm.) : h″ S′ = (15.4983 mm.) : : 1.3365 : 1.0000. Since then 1. and .3365 are proportional and express the relation of the sines in the case, it can be formulated, from what has been shown that, 15,4983 : 1.0000 : : .3365 : .02171 mm. ; or the height of the image at S″ produced from an object .3365 mm. at S′. Hence the rule to find the value of S″: multiply the height of the object at S′, by the relative refraction within the eye, and divide the product by the anterior focal distance and the quotient will be the height of the image at S″. Thus we find the sine in the normal eye is, by this calculation, .02171 mm., which closely agrees with .0218 mm., found by experiment, as the sine of the smallest angle, under which the several parts of an object can be seen under an angle of 60″. If, .02171 mm. be the value of S″ what will be the altitude of the types at 5000 from S′ or (5000 + 20.7135 =) 5020.7135 from k? Formula: 15.4983 : .02171 : : 5000 : 7.0330 mm.; hence this estimate corresponds with that of the emmetropic types as found by another method of calculation heretofore given.

Ametropia may be estimated in the same manner by the rule just given. Take a case of hypermetropia equal to 20 dioptrics. Here (Fig. 6) h S''_4 = 15.3882 and h S′ = 15.4983 mm. The refraction in such an eye is but .9928, (see appended table) and the defect of refraction is .3437. That is if the refraction in the air were, .9928, the relative refraction within the eye = .3437. Hence according to the above rule the formula; 15.4983 : .9928 : : .3437 : .02202 mm., which latter quantity is the value (altitude of the image) at S_4 of the case. It follows as 15.4983 : .02202 : : 5020.7135 7.8334 mm., the types required at 5000 mm. from S′ in the case. If now the refraction be aided by the application to the eye of a + lens of 50 mm. principal focal distance with its 5′ placed 13.7451 mm. from k, the refraction will thus be increased from .9928 to 1.3365, or by the refractive deficit — .3437 —, and the image will thus be decreased at S''_4 to .0215, k will become 5, and the types at 5000 mm. from S′ will be 7.0330 in altitude. The latter are not large enough, and must be increased to enable the several parts of S''_4 in the case to be seen under the smallest angle (60″) the eye is capable of viewing objects distinctly. If these types be increased to 7.1334 mm., then S''_4 will become .2171 omm. and be clearly visible.

Thus while we have been aiming to see objects under an angle of 5′, in theoretically employing the constants of Stammehaus and Heyl, in the simplified diagrammatic eye of Helmholtz, it is seen that the angle is 4′ 48″.

It is further observed that when the correcting lens is applied and the image at the retina has the same size, the angle in the ametropic, does not correspond with that in the emmetropic eye. It would seem therefore in determining the size of the types to be employed in estimating and correcting ametropia, these should be based, not upon the visual angle under which the object is seen, but upon the height of the image at the retina and the ametropic refraction.

It will also be noticed that under the new constants, the types will be smaller that those of Snellen, and hence those of that authority, must be reduced in order to correspond with the types dictated by the new constants. This is in harmony with the well known fact, that there are persons whose visual acuteness is greater than denoted by the types of Snellen.

Finally the position of the correcting glass may be found by refraction and proportion. Since the focal distance of such a glass must be equal to the distance from the object (Fig. 6) o to k, + the distance from 5 to h_2 of the lens, equal in this case (hypermetropia = 20. dioptrics) 50 + 2.7500 (the posterior locus) = 52.7500 mm., the proportion of its radius, or focal power employed, in correcting the ametropia, may be estimated from the defect of refraction in the case; viz: .3437. Hence the formula: 1.3365 : .3437 : : 52.7500 : 13.5654 mm., which latter quantity equals the distance from k to 5′ of the lens. This closely corresponds with this distance found by another calculation or 13.7451 mm. The difference between the two quantities being but .1797, it is probable, were all the calculations made with a greater number of decimals, these distances would be still nearer alike, practically they may be assumed as equal, differing but .1796 of a mm.

In the appended table will be found the Dioptrics employed in determining ametropia, with their value in the metrical system and in English

inches; also the anterior and posterior focal distances of the eye in the normal state and in the several grades of ametropia, as well as the normal and abnormal indices of refraction of the eye.

Zanesville, Ohio, May 22d, 1882.

TABLE OF THE INDICES OF REFRACTION OF THE EYE.

Number Dioptrics.	+ or —	Distance of object in mm.	Distance of object in Eng. inches	Anterior focal distance, mm.	Posterior focal distance, mm. (Conjugate).	Ametropic Re.	C Exercised.	Sum of Re + C	Defect of Re	Excess of Re.	C not exercised.	Emmetropic Re.	Emmetropic C.
00.0	o	5000.	200.0		20.7135	None	None	1.3365	None	None	None	1.3365	o
14.0	+	84.6	2.8		17.3301	—	1.1181	1.1181	None	None	—	—	1.1181
								2.4546					
20.0		50.	2.		15.3882	0.9928		2.1109	.3437				
18.0		55.5	2.2		15.8321	1.0215		2.1396	.3150				
16.0		62.5	2.5		16.3028	1.0519		2.1700	.2846				
14.0	+	70.	2.8		16.7148	1.0784		2.1865	.2581				
12.0		83.3	3.3		17.2831	1.1151		2.2332	.2214				
10.0		100.	4.		17.8025	1.1486		2.2667	.1879				
9.0		111.	4.4		18.0666	1.1657		2.2838	.1708				
8.0	Hypermetropia	125.	5.		18.3405	1.1833	1.1181	2.3014	.1532	None			1.1181.
7.0		142.8	5.7		18.6164	1.2011		2.3192	.1354				
6.0		166.6	6.6		18.8986	1.2193		2.3374	.1172				
5.0		200.0	8.		19.1868	1.2386		2.3567	.0979				
4.0		250.	10.		19.4801	1.2569		2.3750	.0796				
3.0		333.3	13.3		19.7791	1 2762		2.3943	.0603				
2.0		500.	20.		20.0843	1.2959		2.4140	.0406				
1.0		1000.	40.	15.4983 mm.	20.3957	1.3159		2.4340	.0206				
20.0		50.	2.		31.6750	2.0437	0 4109			.7072	.7072		
18.0		55.5	2.2		29 9467	1.9322	0.5224			.5957	.5957		
16.0		62.5	2.5		28.3924	1.8319	0.6227			.4954	.4954		
14.0		70.	2.8		27.2269	1.7567	0.6979			.4202	.4202		
12.0		83.3	3.3		25.8427	1.6674	0.7872			.3309	.3309		
10.0		100.	4.		24.7624	1.5977	0.8569			.2612	.2612		
9.0		111.	4.4		24.2691	1.5659	0.8887	2.4546	None	.2294	.2294		
8.0	Myopia —	125.	5.		23.7918	1.5351	0.9195			.1986	.1986		
7.0		142.8	5.7		23.3429	1.5061	0.9485			.1696	.1696		
6.0		166 6	6.6		22.9140	1.4784	0.9762			.1419	.1419		
5.0		200.0	8.		22.5040	1.4520	1.0026			.1155	.1155		
4.0		250	10.		22.1136	1.4268	1.0278			.0903	.0903		
3.0		333.3	13.3		21.7404	1.4027	1.0519			.0662	.0662		
2.0		500.	20.		21.3832	1.3797	1.0839			.0342	.0342		
1.0		1000.	40.		21 0413	1.3588	1.0958			.0223	.0223		

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